Chapter 2: Newton on the ball

It can be said that in terms of popularity, football holds the title of king of sports. It is the medium that unites millions of people around the world, often determining their everyday mood and emotions. It is quite telling that FIFA, the International Federation of Association Football, has 209 members, whereas the United Nations just 193[4]. How many of us have not woken up one morning feeling grumpy just because the night before our favorite player missed a crucial penalty kick during an important game? Reading the sports newspapers the next day, we may find out that the reason that the ball did not end up in the net is because the player is not on form, or even more scientifically, that he is just unlucky when he takes crucial penalty kicks for his team.

On the other hand, Newton certainly holds the title of king of classical physics. He was the first to show us that the same physical laws that define the drop of an apple also determine the motion of planets million of kilometers from our Earth; a truly remarkable leap in human knowledge.

How many of us, even those with some scientific knowledge, have thought that the real reason that the ball hit the post, may have more to do with Newton's laws than with the bad luck of a football player? How many of us are aware that if we know the magnitude and direction of the force exerted on the ball, we can use mathematical equations to determine the exact position at which it will end up?

<u>Newton's Laws</u>

Newton's famous three laws of motion are stated as follows:

- A body that is at rest or is moving with a constant velocity will remain in this state as long as no external force is acted upon it.
- For a body with a constant mass, the total force acting on it is equal to its mass (m) times the acceleration (a) that it obtains, i.e.,

$$F = ma$$

- If a body exerts a force (action) on another body then the second body will also exert a force (reaction) of equal magnitude and opposite direction on the first one.

By using football as our case study, we can present all three of the above laws. When a ball is set for a free kick, it will remain stationary until a force is applied from the foot of a player (1^{st} law). When the foot exerts the force, the ball will also apply an equal and opposite force on the foot (3^{rd} law).

Finally, the 2^{nd} law exhibits itself as follows: Once the ball leaves the foot, two forces are exerted on it during its flight; its weight (vertically downwards) and the air resistance (opposing its motion). If there is also spin applied, then a third force appears, the so called *Magnus force*. This has a direction dependant on the direction of the spin (clockwise or anticlockwise) and is perpendicular to the axis of rotation of the ball. We will discuss the air resistance and Magnus force in more detail in later chapters. The point is though, that according to the 3rd law, the total effect of all three forces will be to provide the ball with an acceleration, in this way also determining its velocity and total trajectory.

The weight force often gets confused with the concept of mass, so it is worth at this point to clear up the issue. Mass is measured in kg whereas weight which is a force, is measured in Newtons. Mass is the measure of resistance of a body to any change in its state of motion and as we have previously mentioned, is a scalar quantity. It is much more difficult to move a body with a large mass than one with a small one. Also, the mass of a body does not change no matter where the body is positioned. Weight on the other hand is the force that is exerted by Earth on the body. Since it is a force, it is a vector quantity with a direction towards the center of the Earth. The formula that gives it is the well known,

$$W = mg$$

Where g, is the acceleration of gravity.

The acceleration of gravity g that a body acquires due to the gravitational pull is independent of the mass of the body but does depend on the geographical position. So for example, at the Equator it has a value of approximately $9.78 m/s^2$, whereas at the poles it is about $9.83 m/s^2$. This means that at the poles the body will weigh around 0.5% more than at the Equator. A typical football has a mass of 0.43kg. At the poles its weight will be 0.43*9.83 = 4.227N whereas at the Equator, 0.43*9.78 = 4.205N. Its weight has decreased but its mass of course has remained the same. If you wish to lose weight you should move to somewhere with a more tropical climate. Unfortunately this will not affect your mass which will not have changed at all.

Returning to the trajectory of the ball, what is important for our analysis is that since it is under the influence of several forces, according to the 2nd law, the ball will acquire an acceleration. Due to this acceleration the velocity and the position of the ball will change. The relative magnitude and direction of the three forces (W, weight, D, air resistance, F_M, Magnus force), will determine the exact form of the trajectory. Since the forces are vector quantities, they can also be represented by arrows in the same way that we showed for velocity in the previous chapter.

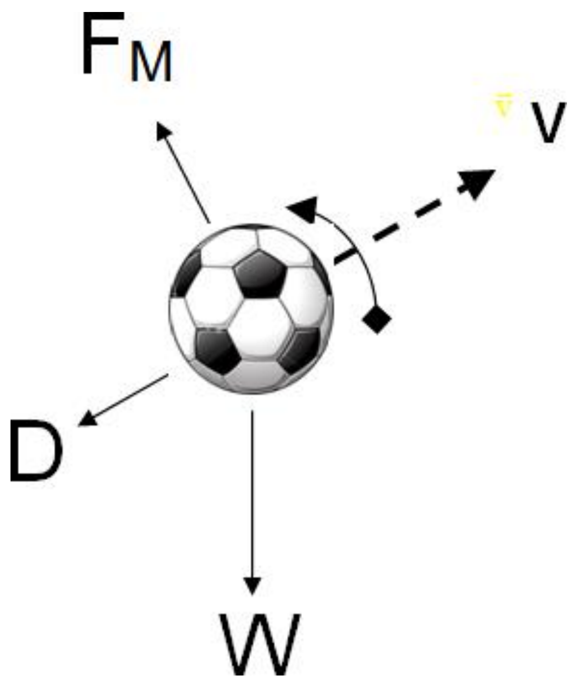

Figure 2.1: Forces acting on a football

In order to calculate the acceleration, velocity and position of the ball at each instant, we must solve a series of complex equations based on Newton's three laws. This is where the shooting ability of each footballer comes into play. The player substitutes the calculations required to solve complex equations with his experience and talent.

As it can be seen in the following diagram, the direction for example of the Magnus force (determined by the footballer's kick), will affect the trajectory of the ball. By giving the axis of rotation the appropriate inclination, the ball can move to the right, upwards or a combination of both. Newton and the footballer are in full cooperation.

Figure 2.2: Magnus force and ball trajectory

A simple example

In order to get an idea of the trajectory the ball can take after a kick, we can look at a simple arithmetic example by making some simplifying assumptions. With Newton's help, we can determine the deflection of a ball due to spin for a free kick from 25m, with an initial ball speed of 30m/s and a spin rate of 7Hz. We will discuss more about rotational motion in the next chapter, just bear in mind for now that a rate of 7Hz implies that the ball completes seven revolutions round itself in each second.

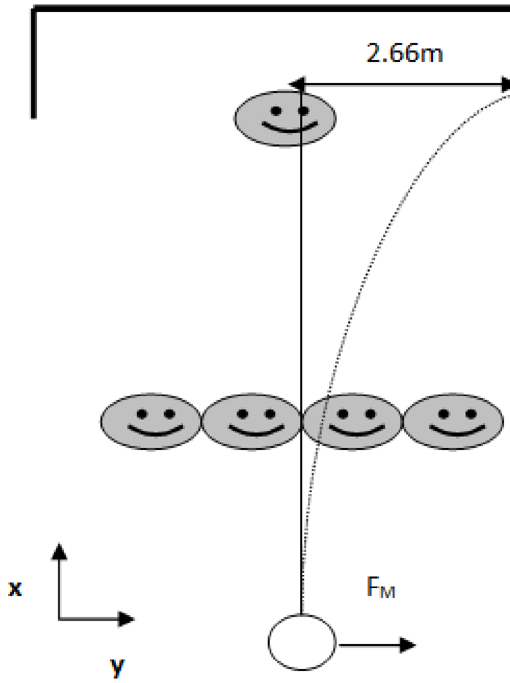

Figure 2.3: Free kick simple example

The assumptions we make are that the motion is confined to two dimensions (let us call them x and y), thus ignoring the height of the ball, and that the air resistance is zero. The air resistance is certainly not negligible but for the scope of our example the error produced by this assumption has only a negligible effect on the result.

Before we start, we must provide a formula for the Magnus force. For a football we can approximate its value with,

$$F_M = \pi^2 \rho R^3 V f$$

Where,

ρ is the air density

R is the radius of the ball

V is the ball speed

f is the ball spin rate[5]

The number *π* (pi), you may already know, is one of the most important mathematical constants and its value is approximately 3.14. The air density *ρ*, i.e. its mass per unit of volume, at a low altitude is roughly 1.2kg/m^3.

So we have,

$$F_M = \pi^2 \rho R^3 V f = 3.14^2 \cdot 1.2 \cdot 0.11^3 \cdot 30 \cdot 7 = 3.3N$$

According to Newton's 2nd law, this lateral force will produce a lateral acceleration,

$$a_y = \frac{3.3}{0.43} = 7.67\frac{m}{s^2}$$

The time the ball takes to reach the goals can be found by dividing the distance *x* from where the free kick is taken to the ball speed[6]. Since the acceleration is assumed constant, according to the formula provided in the previous chapter we have,

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}a_y\left(\frac{x}{V}\right)^2 = \frac{1}{2}\times 7.67\times\left(\frac{25}{30}\right)^2 = 2.66m$$

So even though significant simplifications have been assumed, our analysis shows that the ball will deflect by 2.66m and will end up in the right hand corner of the goal. The cooperation between player and Newton has been successful.

<u>A more detailed analysis</u>

If we wish to perform a more detailed and precise analysis, we must fully solve the complex equations derived from the application of Newton's laws. Using suitable software, we can simulate the motion of a football for important aspects of the game, such as free kicks outside the penalty box and corner kicks. The technique that is evoked is similar to that employed by flight simulators and in simple terms can be described as follows. At each time instant we calculate the forces acting on the ball. From the forces, and by use of the 2nd law, we can estimate the acceleration. By knowing the acceleration at time *t*, we can estimate what the speed and the position will be after time *δt*, as long as *δt* is small enough (certainly smaller than 1 second). So basically, by determining the position and velocity of the ball at any given instant, we can calculate its new position and velocity at a time point a little later. By progressing in this way we can determine the whole trajectory.

We begin our simulation[7] with a free kick just outside the penalty box, 20 yards (18.28m) from the goal line. If the initial speed of the ball is 25m/s, the spin rate is 7Hz and the initial inclination of the ball trajectory is 17 degrees, we see from figure 2.4 that the ball ends up in the top left corner of the goal.

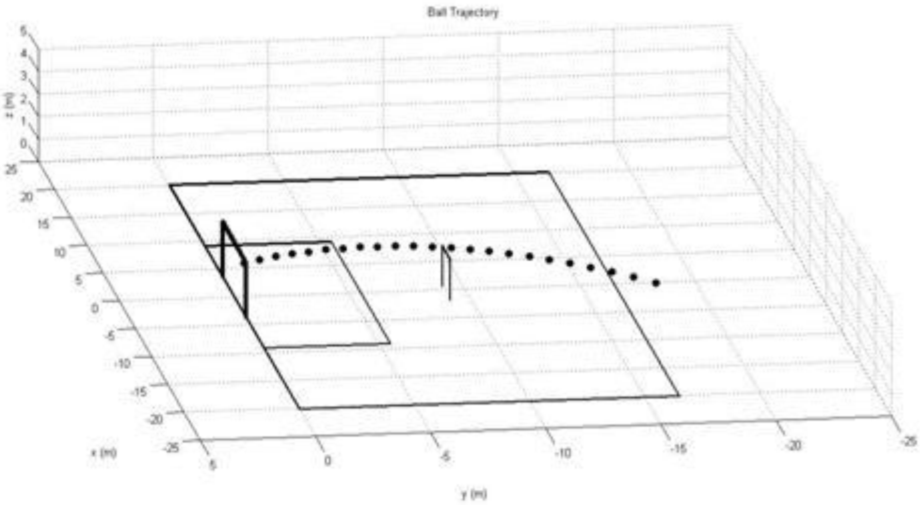

Figure 2.4: Free kick with 17 degrees inclination

For the next kick, we just increase the inclination by one degree, to 18 degrees. If we do this, the ball hits the bar.

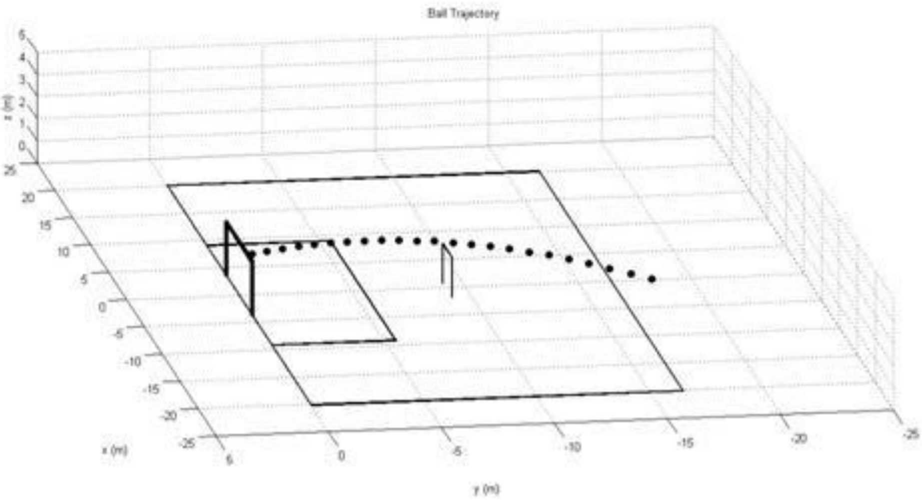

Figure 2.5: Free kick with 18 degrees inclination

Finally, if we reduce the inclination to 16 degrees, the ball hits the defenders' wall (positioned at a distance of 9.15m as determined by the rules).

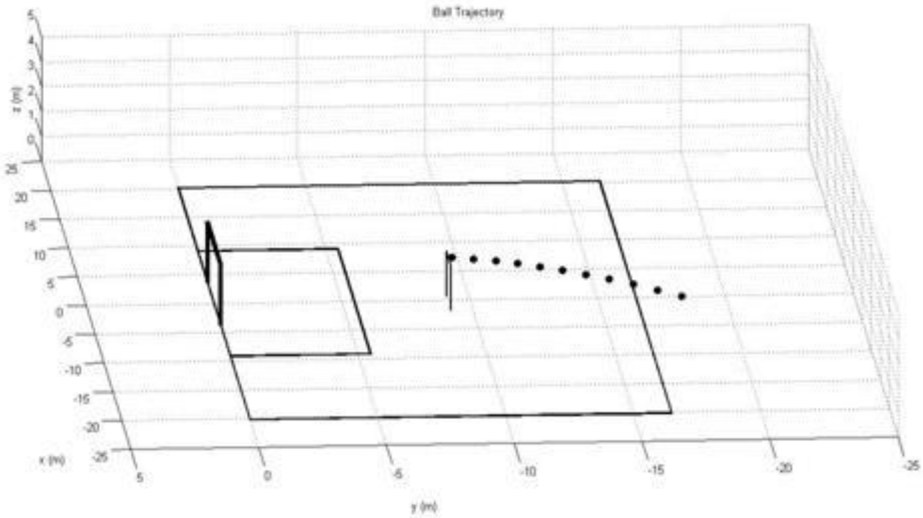

Figure 2.6: Free kick with 16 degrees inclination

It is very interesting that only a slight difference in the initial inclination given by the player modifies the final result by so much. Free kicks must be taken with an accuracy of 1 degree! In fact, by use of appropriate mathematical calculations, it can be shown that for the free kick simulated here, an error of 1 degree will produce a difference of 32cm to the height the ball will have when it crosses the goal line.

One of the most spectacular (and rare) occasions of goal scoring is by a direct corner kick. With our mathematical simulation, instead of defining the initial conditions of the kick (for example initial velocity, spin, etc), we can define the point at which we wish the ball to end up at and then get the computer to calculate the required initial conditions. So by setting the top right corner as our aim, our model finds the velocity, spin, etc., required. Of course there are an infinite number of combinations for which the outcome will be the same. In any case, the result (even at the level of a mathematical simulation) is truly impressive!

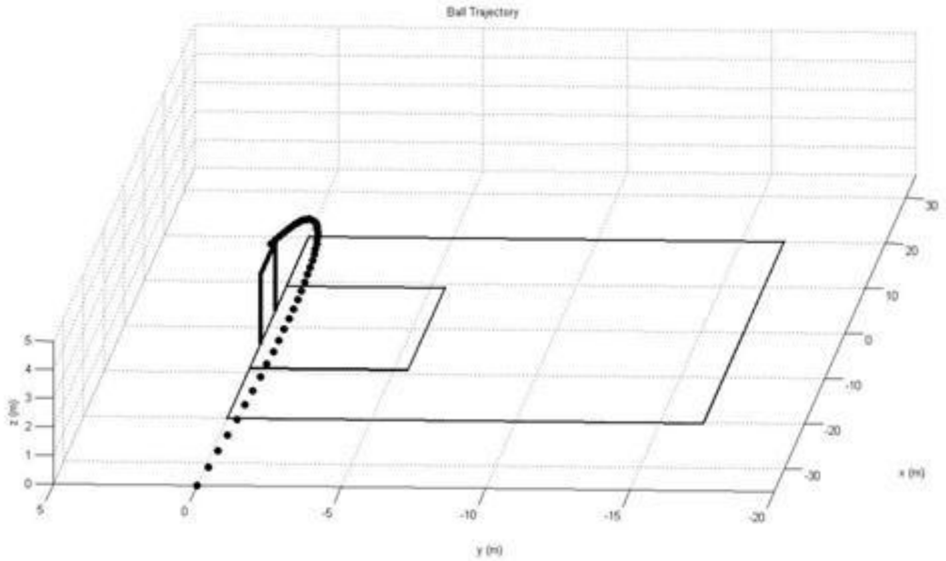

Figure 2.7: Goal scored from direct corner kick

Momentum

The *momentum* of a body is defined as the product of its mass times its speed. So for example, although the maximum speed of the shot in the shot put event is about half that of a football, its mass is sixteen times larger. The outcome is that the momentum of a shot is eight times that of a football. Momentum is a vector quantity and its direction is that of the velocity of the body.

Whenever a force is applied to a mass its momentum changes. Newton's 2nd law, in its more general form, can be expressed in relation to the change in momentum as follows: The force acting on a body is equal to the rate of change of its momentum.

The corresponding mathematical formula is,

$$F = \frac{\delta J}{\delta t}$$

Where δJ the change in momentum and δt is the time interval over which the force is applied.

For bodies with constant mass, this relationship is equivalent to our familiar $F=ma$, as the rate of change of momentum is equal to mass times the rate of change of speed, i.e. acceleration. The general form of the 2nd law is often more useful than the one containing the acceleration. If for example we drop a basketball, we can quite easily calculate the average force acting on it while it hits the ground.

Fontanella[8], has performed such experiments and measured the speed of a basketball before and after impact with the ground as 4.5m/s and 3.5m/s. A men's basketball has a mass of 0.61kg, so its momentum before and after impact is 4.5*0.61 = 2.745Ns and

3.5*0.61 = 2.131Ns, respectively. Since the velocities have opposite directions, we can easily find the total change in momentum by adding the two[9], as 4.88Ns.

Fontanella also measured the time of contact of the ball and the ground as 0.016s. So the average force acting on the ball, according to the general form of the 2nd law, is 4.88/0.016 = 305N. By ignoring the weight of the ball (which is much smaller), we see that the ground exerts a force on the ball that is approximately 300N. This is the average force; Fontanella also measured the maximum force as 650N. So a basketball also exerts (as a consequence of the 3rd law), a force on the ground that is roughly 100 times its weight, about the weight of an athlete. This is possibly the reason, that when playing this game (at a younger age), my fingers were constantly in pain.

Newton's 2nd law also tells us that for the same change in momentum, the acting force is smaller when the time interval over which it acts is greater (dividing by a larger number will produce a smaller result). This is exploited by athletes performing high jumps, for example a basketball player performing a "slam dunk". By bending his/her legs when landing, he/she will increase the time of contact with the ground, so decreasing the maximum force exerted on the legs. If the legs were kept straight, the team medics would have a lot of work to do.

Another consequence of the 2nd law is that if a body or system (group) of bodies does not have any external force acting on it (or the sum of forces is zero), then the total momentum remains unchanged. This principle is known as the *Principle of Conservation of Momentum*. To obtain an understanding, let us look at the sport of shooting. Before an athlete shoots with his/her rifle, the fact that the rifle is stationary means that its momentum is zero. As soon as a shot is taken, the bullet starts travelling in the forward direction and acquires a momentum in the same direction. In order for the total momentum to remain zero, a momentum must be created that is in the opposite direction. This is exactly the reason that we get the familiar rifle recoil.

We can use the Principle of Conservation of Momentum to estimate the consequences of a collision in sports. Let us for example, look at ice hockey, a particularly fast sport where the speeds reached by players can be as high as 40km/hr. Let us assume that one of them has a mass of 80kg and is moving towards the left with a speed of 9m/s and another with a mass of 90kg is moving in the opposite direction with a speed of 8m/s. Let us also assume that the athletes collide an essentially become a unified mass (something not particularly comfortable for them). What will their common speed be after the collision?

The athlete that is moving to the left will have a momentum of 80*9 = 720Ns. The opponent moving to the right will have 90*8 = 720Ns, i.e. the two momentums are equal. As we have already mentioned, since momentum is a vector quantity, and the athletes are moving in opposite directions, in order to find the total momentum before the collision we must subtract the two, which gives us a result of zero. As no external forces are acting on the players[10], the total momentum after the collision will remain zero. The only way for this to happen is for the two athletes locked together to remain stationary.

Coefficient of restitution

A basketball bounces more than a golf ball, the reason being that when colliding with the ground it suffers fewer energy losses. We will talk more about the concept of energy in a later chapter. At this point, we will determine the percentage of speed that the ball retains after the collision by use of the *coefficient of restitution*,

$$e = \frac{V_{after}}{V_{before}}$$

Where, V_{after} and V_{before} are the speeds before and after the collision.

The larger this parameter is, the more *elastic* the collision, i.e. the fewer the energy losses. Furthermore, if we let a ball drop from a certain height, the height that it will reach after the rebound is higher for balls with a greater coefficient of restitution. More specifically, it can be proved that the coefficient of restitution is approximately given by,

$$e = \sqrt{\frac{h_{after}}{h_{before}}}$$

Where, h_{after} and h_{before} are the heights after and before the collision.

The symbol $\sqrt{\ }$ is that of the *square root*, which is the number that when multiplied by itself will give the original number. So for example, the square root of 9 is 3.

For some sports, the heights at which the balls must rebound to are strictly defined. So in basketball, according to the International Basketball Federation (FIBA), if a ball is dropped from 1.8m it must return to a height between 1.2m and 1.4m. From the above formula we can deduce that the coefficient of restitution will lie between,

$$e = \sqrt{\frac{1.2}{1.8}} = 0.82 \text{ and } e = \sqrt{\frac{1.4}{1.8}} = 0.88$$

In the same way, if a tennis ball is dropped from a height of 100 inches (254cm) on to a concrete floor, it must rebound to a height between 53 inches (134.62cm) and 58 inches (147.32cm). So the limits are,

$$e = \sqrt{\frac{53}{100}} = 0.73 \text{ and } e = \sqrt{\frac{58}{100}} = 0.76$$

A basketball certainly bounces better than a tennis ball.

It is important to note that the coefficient *e*, depends not only on the type of ball but also on the properties of the ground. This is why the above limits are defined with respect to a concrete floor, as the tennis ball will certainly bounce differently on clay and on grass.

Another parameter that affects the value of the coefficient is temperature. If you are a golf player, it may be interesting to know that the coefficient of restitution of the golf ball drops from about 0.80 to 0.67 when it is cooled. It would make sense then, on a very cold day to keep the ball in your pocket, in order to maintain it at a higher temperature. This may also be the reason that short distance runners are supposed to perform better on warm days. At a higher temperature, the coefficient of restitution between shoe and ground will increase, thus helping the athletes.

Measuring the coefficient of restitution can constitute a simple physics experiment that you can plan yourselves. By dropping different types of balls you are able to perform calculations similar to the above. You can also realise the following simple study. In tennis rackets there exists a certain point from which, when the ball hits it, it rebounds with the highest speed. This point is called the *power point* or many times the "sweet point" [11] due to the nice sensation the athlete gets. By fixing a racket's arm to a table and letting its head be on the side you can drop a tennis ball from a suitable height (for example 0.5m) and then note how high it will rebound. You then repeat the process by letting the ball drop onto different points on the racket head. The point with the best rebound is the power point.