Chapter 1: Incredible speed races

High speeds and accelerations fascinate most of us. The feeling we get when an airplane takes off or when we start a ride on a roller coaster, is surely unique. In the case of sport, when we are referring to high speed, there is one athlete that immediately comes to mind: Usain Bolt! By carefully analysing his incredible performance, we will understand more about the physics involved in setting his amazing records.

A few thoughts on speed and acceleration

We live at a time when speed rules our lives. Our everyday routines depend on it. Almost all of us that work in big cities feel our alarm clock like the starter pistol initiating our daily schedule. Our first mission is to arrive at our work before our boss shows us a.. yellow card. So if for example we are living somewhere that is 15 minutes by car from our office, what is the speed that we need to be going at in order to avoid the caution from the boss?

The answer to this may look simple but it actually requires a bit of thought. Most of us are aware of the fact that in order to calculate speed, we must divide distance by time. In physics we say that speed is given by the formula,

$$V = \frac{S}{t}$$

Where, *S* is the distance and *t* is the time.

If we assume that the road from our home to our office is 7.5 kilometers long, then the answer that comes naturally to mind is that we must drive at a speed of 7.5km/0.25hrs, i.e. 30km/hr. The terms km/hr are the *units* of speed that we have used in this example. As we will see, physicists are often more comfortable with units that we are not used to in our everyday dealings.

If you look of course at the speed indicator, it will rarely show this value, i.e. 30km/hr. Once you are on a high speed road for example, you may reach 70km/hr. On the other hand, as we approach the centre, pedestrians will probably overtake you, while you are stuck at some traffic light.

The key here is the difference between *average* and *instantaneous* speed. The former is found by dividing the *total* distance covered by the *total* time taken. On the other hand, the instantaneous speed (i.e. that shown on the indicator), reveals how fast we are travelling at that specific instant. It also, results as the quotient of distance over time, the difference being that it is not the total time but a very small increment measured at that particular instant. The smaller this time increment is, the closer our result approaches the instantaneous velocity. In physics we say that the time increment *tends* to zero, i.e. becomes as small as possible. So for example, if during our trip to our office we had only used the time we were on the fast lane, we would have calculated a speed value much closer to the instantaneous speed at that moment.

The average speed, as the term implies, can be viewed as an average value of all the instantaneous speeds. Moving to the world of sport, when Usain Bolt broke the 100 meters world record in Berlin, in 2009, his time was 9.58 seconds. How fast did Bolt run at this race? A first approach would be to estimate his average speed. By subtracting his reaction time, which for that race was 0.146 seconds, we find that the actual time taken to cover the 100 meters was 9.434s (seconds). It follows, that his average speed was 100/9.434, which is 38.15km/hr. So by comparing average speeds, Bolt travelled faster than your car did. Of course the car covered 7.5km, whereas Bolt stopped after 100 meters.

More interestingly though, what was his instantaneous speed during the race? The most accurate method is to use the, so called, split times, i.e. the times taken to cover each 10 meter interval. The following table obtained from the IAAF (International Association of Athletics Federations) can be used for this.[1]

Position (m)	Time (s)	Speed (m/s)
10	1.89	5.73
20	2.88	10.10
30	3.78	11.11
40	4.64	11.63
50	5.47	12.05
60	6.29	12.20
70	7.10	12.35
80	7.92	12.20
90	8.75	12.05
100	9.58	12.05

Table 1.1: Split times of Usain Bolt when he broke the 100m world record

The values in the third column are derived by dividing the 10 meters of each split by the time that is needed to cover it. For example, for the last 10 meters, we find the answer, 10/(9.58-8.75) = 12.05m/s. As the time increments are now much smaller, the speeds that we calculate are closer to the instantaneous speed of the athlete. In reality of course, these are the average speeds for each split. In any case though, they still provide a very useful indication of the athlete’s performance. By use of this approach we deduce that the maximum speed is 12.35m/s, just over 44.46km/hr. Bolt may find it difficult competing against your car in the fast lane but he would have a good chance if you were driving a small motorbike. Interestingly, if he managed to maintain this speed throughout the race, he would clock a time of 100/12.35 = 8.1s.[2] This of course would be not possible as his speed is not constant but varies according to the value of his *acceleration*.

The characteristic feeling we get in our stomach when we travel at high speed is not a result of the speed itself, as it is sometimes wrongly believed. In reality, what increases our adrenaline is our *acceleration*, i.e. the variation of speed with time. This can easily be proven as follows. During the takeoff phase of a typical airliner, the aircraft will start of from rest and reach a speed of approximately 250km/hr (i.e. 70m/s) until it lifts off the ground. If you look at your watch (I have done this several times), you will find that the time taken is about 30 seconds. The speed has changed by 70m/s in 30s, so the aircraft is moving with an average acceleration of 70/30 = 2.33m/s^2. For a fighter pilot this acceleration would seem negligible since during certain maneuvers, he/she will experience accelerations much higher than that. Even so, for us, the sensation is quite remarkable, especially if we are flying for the first time.

Before continuing it is worth mentioning the units of acceleration that were given as m/s^2. These units are defined as meters per second squared. A quantity is “squared”, when it is multiplied by itself, i.e., 1s^2 = 1s*1s. If it is a number, we simply multiply it by itself, i.e. 8^2 will give us 64.

So what happens during the rest of your flight? If you have the chance to fly in an aircraft with entertainment screens, you'll notice that you are provided with information on the flight details, such as speed, altitude and temperature that is often more interesting than the movies shown. You will find that after reaching the predetermined altitude, the aircraft will maintain a constant speed of around 800km/hr, more than three times the takeoff speed. Is the feeling you get three times more intense? Probably not, as you are about to fall asleep, because (apart from the slight turbulence), you do not feel much at all. However high it may be the speed is not changing so the acceleration is zero, and the means of adrenaline stimulation remains inactive.

Wrapping up our thoughts on speed and acceleration, imagine you are next to a train, at a station platform, with other trains sitting next to yours. At some point, you look out of the window and are sure that your train is moving. After a few seconds you realise that you are actually still sitting at the platform. What has happened?

The illusion that you had of commencing your trip, is due to what is termed as *relative speed*. To understand this, assume that as you are travelling by car on a fast lane, a police car is moving in the opposite direction, slowly at a speed of 30km/hr (see figure 1.1). At some point the police signal to you to stop and inform you that according to their radar you were travelling at 100km/hr. You insist that your indicator was showing 70km/hr. Who is telling the truth?

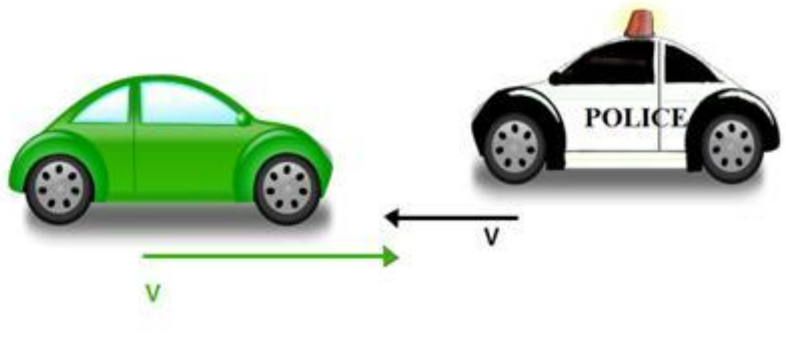

Figure 1.1: Relative speed

What has happened is that the relative speed between you and the road is of course 70km/hr. Relative to the police car though that is moving towards you, you are moving with 70+30 = 100km/hr. When two objects are moving in opposite directions, their relative speed is the sum of their individual speeds. When the motion occurs in the same direction, we subtract the two speeds. Your only chance of avoiding a penalty is if the policeman on duty happens to have some knowledge of basic physics.

Returning to the station platform, what has happened is that the train next to yours has started to move, so although relative to the platform you have remained stationary, you have obtained a relative speed in relation to the other train. This has given you the wrong impression that you are moving and that your journey has started earlier than expected.

From figure 1.1, another important fact can be observed. Many physical quantities, apart from the value they take (in physics we call it *magnitude*), also have a certain direction. These quantities are *vector quantities*, and can be represented as arrows. The size of the arrow is proportional to the magnitude of the quantity. In our case, the arrow denoting the motion of the police car is less than half the size of that denoting your motion. The direction of the arrow on the other hand, portrays the direction of the physical quantity. It should be noted that in actual fact, when physicists refer to speed, they are only referring to a magnitude. The associated vector quantity (with a magnitude and a direction) is referred to as the *velocity* of the object. Other vector quantities are force, torque and impulse, more on those in later chapters. Quantities such as mass on the other hand, that can be fully defined by their magnitude, are not vector but *scalar* quantities.

<u>The race of the century: Usain Bolt vs Airplane</u>

As we previously discussed the speed and acceleration performance of an aircraft during takeoff, it would be interesting to see how these compare to the abilities of our athlete. For this, we need to create two mathematical models, in other words a set of equations that represent, at least to a certain degree, the performance of the athlete and of an aircraft during the take off phase. We can then simulate a race in order to find out which of the two, the fastest athlete in the world or one of our greatest technological achievements, will come first.

The mathematical modeling, i.e. deriving the mathematical equations that represent the model, is quite complicated, so I will not go into it in detail. The same applies to the models presented in later chapters of the book. What I should point out, is that for an airplane, I chose the Northop T-38 Talon, twin engine, supersonic, trainer jet. This was the first supersonic trainer and one of the most popular ever, so we are certainly dealing with a tough competitor.[3]

The fastest sprint races are those of the 100 meters and 60 meters. We will compare the performance of the two competitors in both distances. The results are presented in the following figures. The dark line represents the aircraft and the lighter one, Bolt.

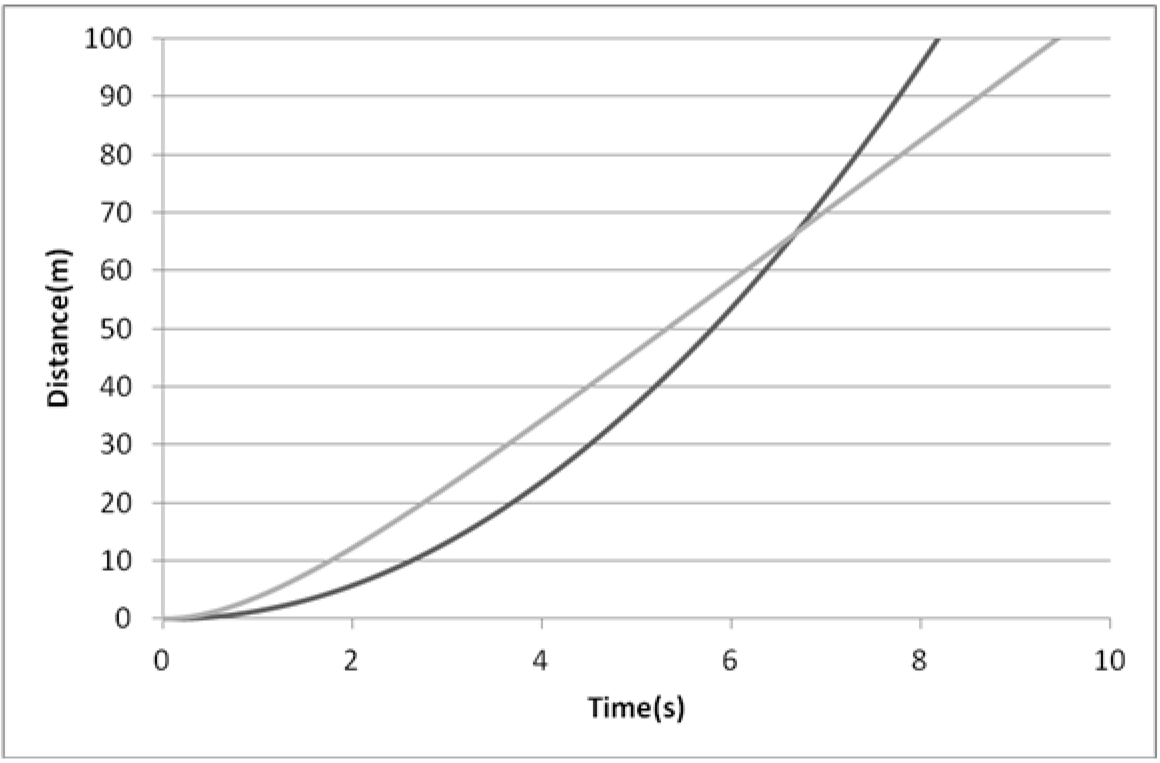

Figure 1.2: Distance comparison

At first glance we notice that the two competitors are very close to each other, especially for the 60m. A more detailed analysis reveals that the airplane wins the 100m, by a margin of 1.3s, whereas our athlete wins the 60m by about 0.2s! The representative of the human race starts off impressively and actually leads the race for the first 66m. It is a great achievement in any case, considering that each engine of the T38 produces a huge amount of thrust.

To fully understand what is happening, we need to study the variation of speed and acceleration (see figures 1.3, 1.4).

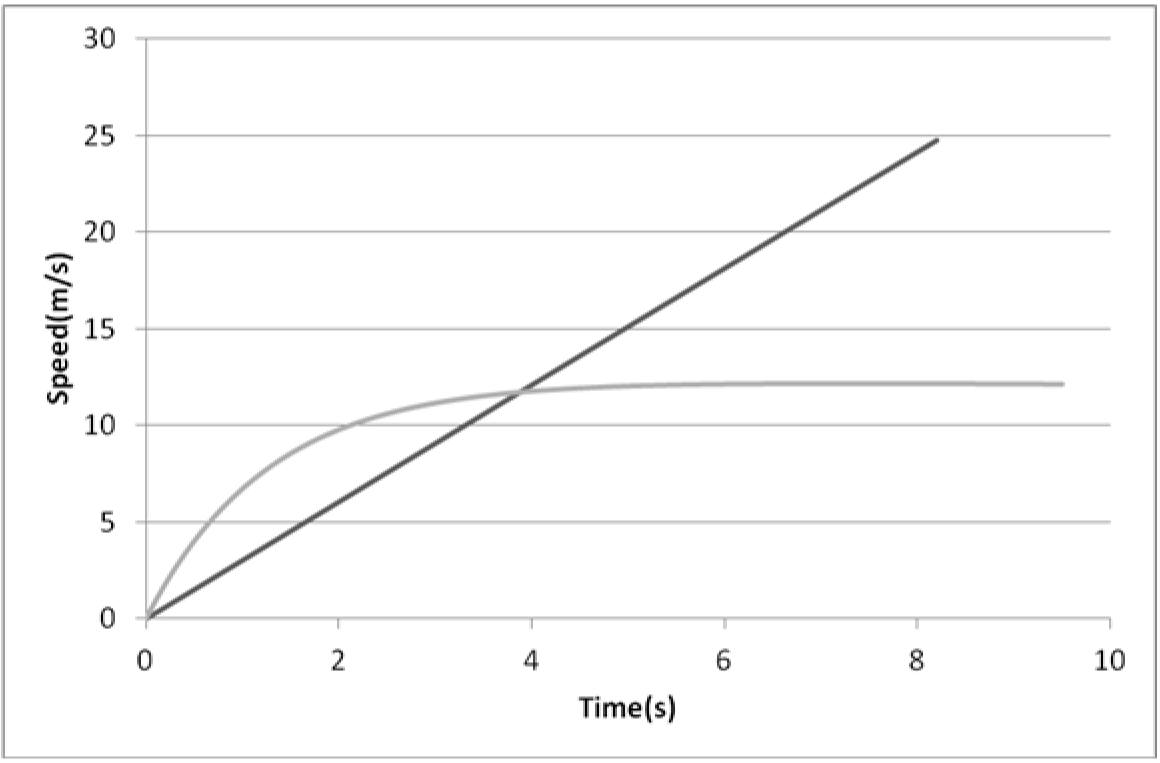

Figure 1.3: Speed Comparison

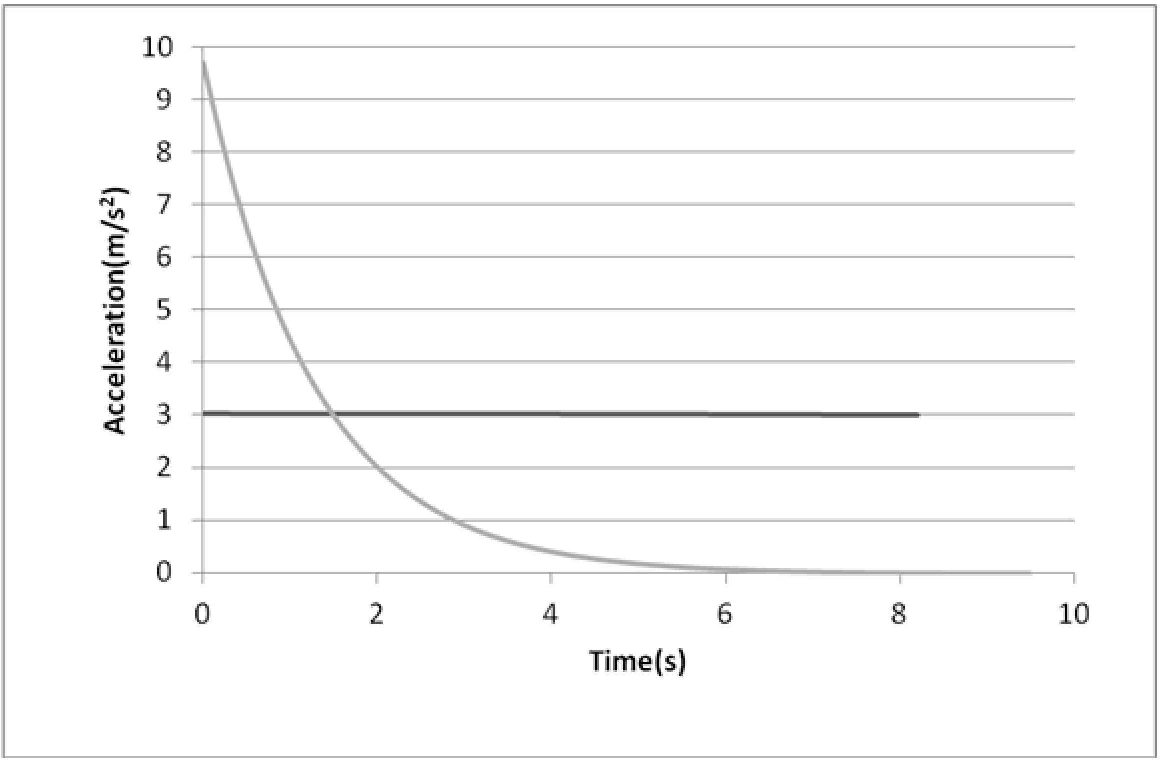

Figure 1.4: Acceleration comparison

From the last figure we can see that the airplane maintains an almost constant acceleration, of just over 3m/s^2 which drops very slightly due to the presence of air resistance (more about this force later). When we move with a constant acceleration the speed increases at a constant rate, as observed in figure 1.3. The final speed of the airplane after 100m is 24.7m/s, almost 89km/hr.

The above could have been deduced from simple kinematics formulas (the field of physics that deals with this type of problem). If we assume the acceleration (*a*) of the airplane to be constant at 3m/s^2, then from physics we know that its speed (*V*) after time *t*, is given by the formula,

$$V = at$$

Also the distance covered in time *t* is,

$$S = \frac{at^2}{2}$$

From this and by use of simple algebra, we can calculate that the 100m are covered by the airplane in 8.16s.

Returning to our race, we see that our athlete makes an explosive start. His initial acceleration is 9.7m/s^2. This is almost the acceleration of someone descending freely towards Earth, so he starts off as if falling from the balcony of a high building! Due to his high initial acceleration his speed increases dramatically and reaches 10m/s (36km/hr), in about 2.2s.

Human power though has its limitations and as we see, his speed reaches a maximum that cannot be overcome (just over 12m/s) and, in fact, it falls slightly during the last few meters. In summary, Bolt starts off with a huge acceleration, more than three times that of the airplane, which is enough to win him the 60m section of the race. On the other hand, his acceleration starts to fall immediately and the aircraft, which maintains an almost constant acceleration, wins the 100m.

So according to our calculations, the race between human and airplane ends in a draw. It should be reminded that the mathematical models used, especially that for the airplane, contain many simplifications and the scope of this example was to comprehend certain principles and not necessarily to announce a winner. The same applies to the rest of the simulations used in this book.

At this point, we can take the opportunity to see how humans would favor against other members of the animal kingdom. The following table is quite revealing.

Animal	Speed (m/s)	Speed (km/hr)
Human	12	43
Cheetah	29	105
Racing horse	25	90
Lion	22	80
Hunting dog	20	72
Cat	13	48
Elephant	11	40

Table 1.2: Speeds of various animals

It can be seen that it would not be wise to take on any of the above; Usain Bolt would find it hard to even surpass an elephant. On the other hand, the sight of an elephant running towards you may be enough to make you break the world record.

Does there exist a speed in nature though that cannot be surpassed, an ultimate, maximum speed? According to Einstein's theory of relativity, the answer is yes. The ultimate speed record is held by light, which in a vacuum travels at 300000km/hr. In the time it takes Bolt to complete the 100m, light can cover 3 million kilometers, i.e. the return journey to the moon, four times. If you have watched athletes compete at a stadium, you may have been under the impression that they start their races before the pistol is heard. This is due to the fact that sound travels much more slowly (approximately 340m/s in air), and so it reaches your ears a little later than the image does. This is also why we see lightning before we hear the associated thunder.

When we are discussing impressive runs, It would of course constitute an omission not to mention those runners that, although do not reach high speeds, are able to complete great distances in very quick times. So for example, if we take the current world record for the men's 10000m, it is 26 minutes and 17.53 seconds, i.e. 1577.53s. From this time we obtain an average speed of 10000/1577.53 = 6.34m/s, almost 23km/hr.

If you were to run at this speed, you could cover 100m in just 15s. Most of us would find this time quite hard to achieve. The athletes competing at 10000m, manage to maintain it for a distance 100 times longer.

<u>Zeno's paradox</u>

It is worth finishing off this chapter by mentioning another imaginary race known as "the paradox of Zeno of Elea", an ancient Greek philosopher of the 5th century BC. The story goes like this:

The tortoise challenged Achilles (the Usain Bolt of his time), to a race, boasting that it could beat him as long as it was given a head start of 10m. Achilles of course laughed, thinking that this would be a walk over. The tortoise though, full of confidence, started to prove to him that he was wrong.

"If I start 10m ahead of you, do you think you will reach that point very quickly?"

"Of course, very quickly indeed", Achilles answered.

"How far do you think I will have moved during this time?" it asked again.

"Maybe one meter", Achilles answered after a bit of thought.

"Very well, so there is still a one meter difference between us. You surely believe that you will cover this distance very quickly, don't you?"

"Exactly!"

“Again though, I will have moved slightly ahead, so a difference will still exist between us, is this not the case?”

“This is the case..”

“So you see that every time you reach the point where I was previously, I will have moved on slightly, and there will always exist an, albeit, small distance between us”.

“Yes, this is true,” replied a disappointed Achilles.

“So you will never be able to reach me and I will have won the race!” said the tortoise triumphantly, offering us hope that we also might be able one day to beat Bolt himself.

This paradox can be redefined as follows. Let us say that we want to go from one side of a room to the opposite one. We will first have to cover half the total distance. After this, once we have reached the midpoint, we will first have to cover the half of the remaining distance, i.e. a quarter of the total distance. Subsequently, when we reach that point, we will again first have to cover half the distance of the remainder, i.e. an eighth of the total distance, etc. This seems to go on forever and as a result we never manage to reach the other side of the room.

The solution to the above paradoxes was provided by mathematical concepts that were developed many centuries after Zeno. What we are actually dealing with is a sum of an infinite series of numbers. By taking for example the second paradox, if the length of the room is 4m, the total distance that we cover is, 2+1+0.5+0.25+0.125+…. Even though the numbers are infinite, it turns out that their sum is not; in this case it is quite obvious that the sum is the 4m that we expect to get. Just don’t try crossing your room using Zeno’s logic, as you may spend the rest of your life doing so! In a similar way, we can prove that Achilles will of course reach the tortoise. As the tortoise though was much cleverer, history has declared it as the undisputed winner of that race.